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# A CORRELATION OF RECOVERY TEMPERATURE DATA FOR CYLINDERS IN A COMPRESSIBLE FLOW AT HIGH REYNOLDS NUMBERS

A. HIRSCHBERG and J. C. VAN MUISWINKEL
Department of Mechanical Engineering, Delft University of Technology, Delft, The Netherlands

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**Abstract**—A correlation is made of recovery factor data for circular cylinders of high thermal conductivity in the stagnation Reynolds number range $10^2 < Re_0 < 10^4$ and Mach number range $0 < M < 5$. The recovery factor is found to be dependent on the Reynolds number in the subsonic region and the data show discontinuities in the transonic region. Empirical formulae are presented for the subsonic and supersonic region separately, as well as an overall formula. The accuracy of the formulae proposed is about 0.2%.

## NOMENCLATURE

$c_p, c_v$, specific heats;
$C_D$, drag coefficient;
$d$, diameter of the cylinder;
$F$, function of Reynolds, $r(Pr, Re)/(Pr)^{1/2}$;
$k$, thermal conductivity;
$Kn$, Knudsen number, $= \lambda/d$;
$M$, Mach number, $= U/(\gamma RT)^{1/2}$;
$Pr$, Prandtl number, $= c_p\mu/k$;
$r$, function of $Pr$ and $Re$, $2c_p(T_m - T)/u^2$;
$R$, ideal gas constant;
$Re$, Reynolds number, $= \rho Ud/\mu(T)$;
$T$, gas temperature;
$T_m$, measured cylinder temperature;
$U$, velocity of the gas.

Greek symbols

$\gamma$, ratio of specific heats, $= c_p/c_v$;
$\eta$, recovery ratio, $= T_m/T_0$;
$\eta^x$, normalized recovery ratio, $= (\eta - \eta_c)/(\eta_f - \eta_c)$;
$\lambda$, molecular mean free path;
$\mu$, viscosity;
$\rho$, density.

Subscripts

$c$, value for continuum flow;
$f$, value for free molecular flow;
0, value at stagnation conditions;
No subscript, = free stream conditions.

## INTRODUCTION

THE HOT wire and the "free" thermocouple are important tools for total temperature measurements in compressible air flow. Examples of the use of free thermocouples are the works of Yanta [1], Behrens [2] and Vas [3]. Figure 1 represents a typical free thermocouple probe. The use of such probes for total temperature measurements is based on an accurate knowledge of the recovery factor $\eta$ of an infinite cylinder of high thermal conductivity placed perpendicular to a uniform flow, as a function of the flow parameters $\gamma$, $M$, $Re$ and $Pr$. The recovery factor is defined as the ratio of the measured cylinder temperature $T_m$ to the free stream total temperature $T_0$ of the gas.

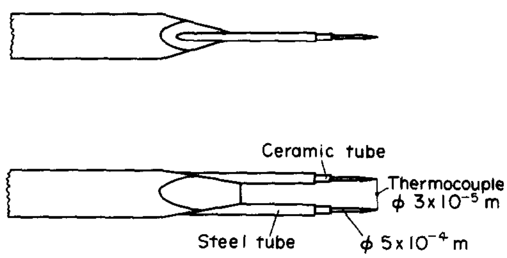


FIG. 1. Typical probe.

Dewey [4] has shown for supersonic flow, that $\eta$ can be calculated from:

$$\eta = \eta_c + \eta^x(\eta_f - \eta_c) \tag{1}$$

where $\eta^x$, the normalized recovery factor, is a function of the free stream Knudsen number $Kn$ only. $Kn$ is defined as:

$$Kn = (\pi\gamma/2)^{1/2}(M/Re. \tag{2}$$

Dewey [4] proposed as a correlation formula for $\eta^x$:

$$\eta^x = Kn^{1.193}/(0.493 + Kn^{1.193}). \tag{3}$$

In order to estimate $\eta$ from formula (1) he used for $\eta_f$ the theoretical value calculated by Stalder *et al.* [5]. For $\eta_c$ he proposed a correlation formula based on his experimental estimation of the hypersonic limit of $\eta_c$, and the data collected by Morkovin [6]. The correlation formula obtained by Dewey [4] for $\eta_c$ is:

$$\eta_c = 1 - 0.05M^{3.5}/(1.175 + M^{3.5}). \tag{4}$$

The nominal accuracy of this formula is 0.4%. The formula should be valid for the full range of Mach numbers from zero to infinity.

In our measurements of the total temperature in a free jet we found significant deviations from the formula of Dewey [4], 2% in the transonic region and 1% in the supersonic region. Extensive measurements were made with thermocouples in a Reynolds number range of $10^2$–$10^4$. The data obtained are in excellent agreement with data collected in the literature.

This investigation showed that at Reynolds numbers higher than 1, the recovery factor in subsonic flow shows a Reynolds number dependence which cannot be described by a rarefaction parameter such as $Kn$. This Reynolds number dependence is a result of the separation of the boundary layer and the change in the structure of the wake as $Re$ increases.

In this paper we shall give our own data as well as the data which we collected from the literature. As a conclusion, correlation formulae will be proposed. The accuracy of these formulae is about 0.2%.

$\eta_c$ at $M = 0.6$. This Mach number was chosen because it is the Mach number at which the maximum data were available and for which the transonic effects are not dominant. Because we did not have a full description of the measurements of Eckert and Weise [10] and Spangenberg [14], we had to make some assumptions in order to estimate the Reynolds number of these data. The Reynolds number of the data of Spangenberg was estimated by supposing that the Reynolds numbers of the measurements with $d = 0.00015$ in were equal to the Reynolds number of the data of Vrebalovich [13]

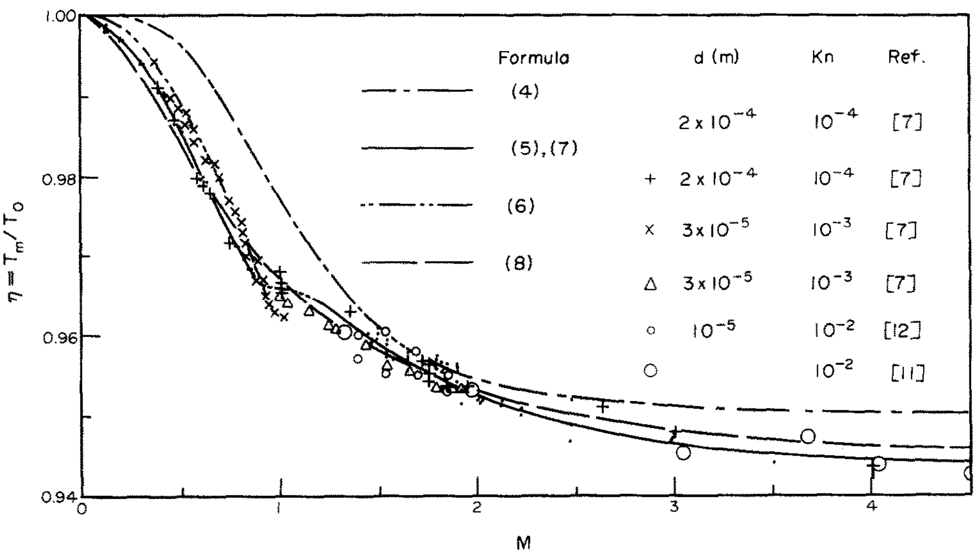


FIG. 2. Comparison of the data with the correlation formulae.

## MEASUREMENTS AND COLLECTED DATA

Our measurements were made in three different types of apparatus:

(a) A free jet with fixed Laval nozzles of 1.5 cm dia. (Mach numbers, $M = 1$, 1.75, 2.05 and 2.65.)

(b) A blow-down tunnel with fixed Laval nozzle ($M = 2.04$), test section $4 \times 4\,cm^2$.

(c) A blow-down tunnel with variable Laval nozzle ($M$ up to 4), test section $27 \times 27\,cm^2$.

$T_0$ was of the order of magnitude of 270 K. Various probes and electronic equipment were used.

Some transonic measurements were made with a wire stretched across the free jet, in order to estimate the influence of the body of the probe on the flow. The effects of end losses were estimated by measuring the temperature of the needles of the probe and using a one-dimensional model like the one used by Yanta [1].

As a check, measurements were made with different length to diameter ratios of the wires, and with an asymmetrical junction, i.e. not in the middle of the wire. Details of the measurements procedure can be found in Hirschberg [7].

The subsonic and supersonic data used to establish the correlation formulae are given in Fig. 2 and Fig. 3. In Fig. 4 the data collected by Stickney [8], Eber [9], Eckert and Weise [10], Laufer and McClellan [11], Vrebalovich [13] and Spangenberg [14] are given. In Fig. 5 a graph is given of the Reynolds dependence of

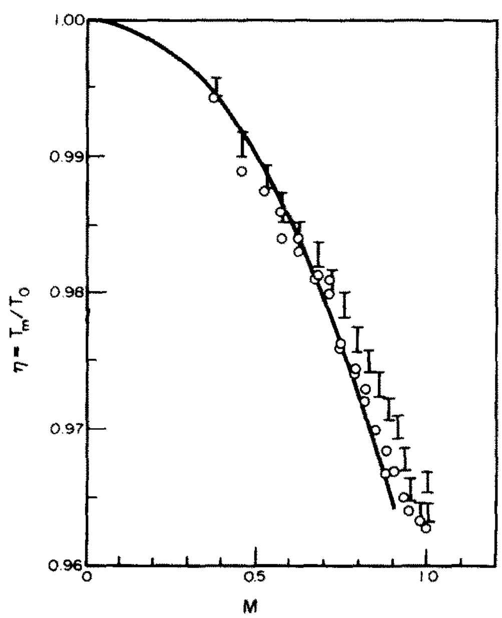


FIG. 3. Subsonic data at $Re_0 \cong 5 \times 10^2$. Influence of the probe: ○—wire stretched across free jet; I—probe in free jet.

for $Kn = 2.5 \times 10^{-2}$ (see Fig. 4). The order of magnitude of the Reynolds number obtained this way is in agreement with the order of magnitude estimated from the data about $\rho$, $M$ and $d$ given in [14].

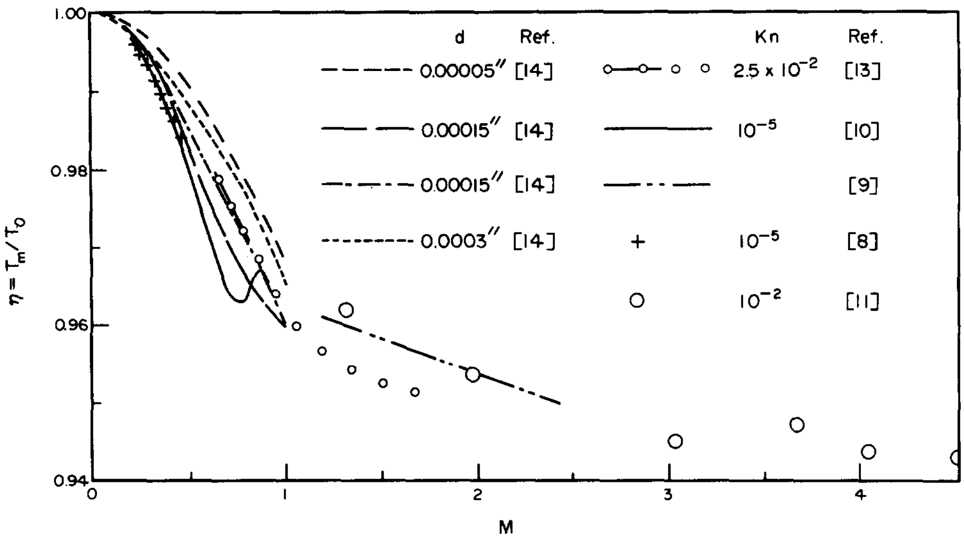


FIG. 4. Data from literature.

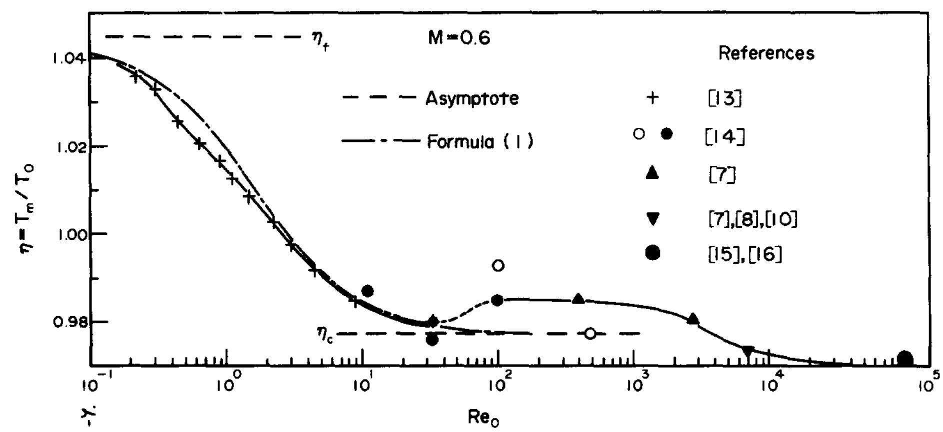


FIG. 5. Reynolds dependence of $\eta$ at $M = 0.6$.

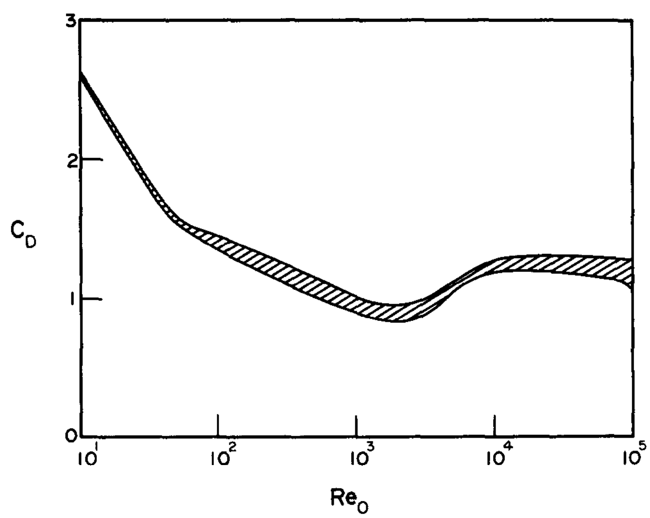


FIG. 6. Reynolds dependence of $C_D$ at low Mach numbers after *M.* Morkovin [19].

The Reynolds number of the data of Eckert and Weise [10] was estimated by comparison with measurements of Hirschberg [7] and Stickney [8]. The value of $\eta$ at Reynolds of the order of $10^5$ was estimated by using local heat transfer and recovery factor data of respectively Schmidt and Wenner [16] and Eckert and Weise [15].

Some of the data of Spangenberg [14] were not considered because of the end losses effect. This explains why we did not use his supersonic data. The subsonic data for the 0.0003 in Pt wire of Spangenberg [14] seems to suffer important end losses. The subsonic data for the 0.015 in Pt wire seems quite unreliable. These two data are represented in Fig. 5 by a ○ instead of a ●.

It is interesting to compare the data of Fig. 5 with mean drag coefficient data (see Fig. 6). From comparison it becomes clear that the "jumps" in $\eta_c$ for $Re = 40$ and for $Re = 5000$ correspond to major changes in the flow pattern (namely the formation of the von Karman vortex street and the appearance of turbulence in the wake). For $10^4 < Re < 10^5$ the recovery factor is expected to be fairly constant. This is due to the fact that the structure of the wake does not show important changes in this region. For Reynolds numbers smaller than 40, the Reynolds number dependence is essentially due to rarefaction effects. As we see from Fig. 5 this effect is correctly correlated by formula (1). We used for $\eta_c$ a value estimated from the value of $\eta$ measured by Vrebalovich [13] at $Kn = 0.025$. The correction of this value was obtained by using formula (1) iteratively.

As a check for the order of magnitude of the hypersonic limit of $\eta_c$, we calculated $\eta_c$ at $M = 5$ from the data of Wagner [17] by using formula (1) in order to estimate the rarefaction effects. We found $\eta_c\ (M = 5) = 0.938$. This value is close to the value deduced from formula (11) (see next section). Details can be found in Hirschberg [7].

## CORRELATION FORMULAE

Because the flow structure at subsonic speeds is essentially different from the one at supersonic speeds, it seems reasonable to use two correlation formulae, one for each region.

We decided to use a power series in $M$ in the subsonic region. In order to select a set of terms as linearly independent of each other as possible we first calculated the correlation coefficients between the powers of $M$. From this analysis it followed that the best results could be obtained with powers of $M^2$ only and that powers higher than $M^4$ were not useful.

For Reynolds numbers of the order of $3 \times 10^3$ we obtained by least squares:

$$\eta = 1 - 0.070M^2 + 0.036M^4 \tag{5}$$

for $M < 1$. The standard deviation is $2 \times 10^{-3}$.

For other Reynolds numbers the data were not sufficient to establish an accurate correlation formula for the subsonic region. We shall establish a formula which is only valid at low Mach numbers ($M < 0.6$). We use as basis the fact that for low Mach numbers and high Reynolds numbers the temperature $T_m$ is expected to be given by:

$$T_m \cong T + r(Pr, Re)\frac{U^2}{2c_p}. \tag{6}$$

Where $r$ is a function of $Re$ through the dependence of the flow configuration on $Re$. This formula, obtained by equating the dissipation in the boundary layer of the cylinder to the heat flux through this boundary layer, is derived by Landau and Lifschitz [18]. Using the definitions for $\eta$ and $M$ and the relation between $T$ and $T_0$ for an ideal gas with constant specific heat ratio $\gamma$ we obtain:

$$\eta \cong \frac{1 + \left(\frac{\gamma-1}{2}\right) r(Pr, Re) M^2}{1 + \left(\frac{\gamma-1}{2}\right) M^2} \tag{7}$$

At low Mach number we can use the following approximation:

$$\eta \cong 1 + (r-1)\left(\frac{\gamma-1}{2}\right)M^2 + (1-r)\left(\frac{\gamma-1}{2}\right)^2 M^4. \tag{8}$$

As a first guess and by analogy with the recovery factor of a flat plate we can try:

$$r(Pr, Re) = (Pr)^{1/2} F(Re). \tag{9}$$

The function $F(Re)$ was estimated from the data collected in Fig. 5.

$$F(Re) = \frac{4.3 \times 10^3 + 0.7Re}{4.7 \times 10^3 + Re}. \tag{10}$$

For $M < 0.6$ and $100 < Re < 10^5$. This formula should only be considered as a first attempt to get an estimation of the Reynolds number dependence of $\eta$ at low Mach numbers.

In the transonic region ($0.5 < M < 1.2$) the data show discontinuities (see Fig. 2). We expect these discontinuities to correspond with major changes in the flow pattern such as the inversion of the pressure gradient and the disappearance of the von Karman vortex street when the flow becomes supersonic. The shape of the curve depends strongly on the Reynolds number (see Fig. 4). The data show an important scatter in this region (up to 0.5%). Since the shape of the discontinuities is not reproducible we decided that it was not useful to compute a correlation function of the data in the transonic region.

At hypersonic speeds one expects the recovery factor to approach a limit asymptotically. For this reason we decided to use in the supersonic region a power series in $M^{-1}$. Further we used the same procedure as for the subsonic case in order to select an optimal set of powers and optimal coefficients. The supersonic data showed a negligible Reynolds number dependence. We obtained:

$$\eta_c = 0.941 + 0.052M^{-2} - 0.027M^{-4} \tag{11}$$

for $M > 1$ and $10^2 < Re_0 < 10^4$. The standard deviation is of the order of 0.2%.

In addition to these two separate correlation formulae, an overall correlation function similar to the one used by Dewey [4] was also estimated by least squares:

$$\eta_c = 1-0.057M^{1.8}/(0.75+M^{1.8}). \tag{12}$$

The standard deviation is of the order of 0.3%. The formula is valid for Mach numbers up to 4 and Reynolds numbers between $10^2$ and $10^4$.

## CONCLUSION

Improved correlation formulae (5) and (11)–(12) for the continuum limit of the recovery factor of an infinite cylinder of high thermal conductivity placed perpendicular to an air flow are proposed. The accuracy of the formulae is 0.2%.

In the subsonic region the recovery factor depends on the Reynolds number [see Figs. 5, 6 and formula (10)].

This Reynolds number dependence seems to be a result of the flow separation and the formation of a turbulent wake for Reynolds numbers increasing from 1 to $10^5$.

Because we expect a discontinuous behavior of $\eta$, for accurate subsonic measurements special attention must be paid to the region of $Re \sim 50$.

As our formulae are based on data below $M = 5$ there is some doubt about the hypersonic limit which we found: $\lim_{M\to\infty} \eta_c = 0.941$, and further measurements would be desirable.

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## DONNEES EXPERIMENTALES SUR LE FACTEUR THERMIQUE PARIETAL D'UN CYLINDRE AUX NOMBRES DE REYNOLDS ELEVES

**Résumé**—Une relation est donnée pour le facteur thermique pariétal relatif à des cylindres circulaires de conductivité thermique élevée, pour un domaine de nombre de Reynolds, aux conditions génératrices, compris entre $10^2$ et $10^4$, et pour un nombre de Mach $0 < M < 5$. On trouve que le facteur thermique pariétal dépend du nombre de Reynolds dans la région subsonique et les données montrent des discontinuités dans la région transonique. On présente des formules empiriques séparément pour les régions subsonique et supersonique aussi bien qu'une formule globale. La précision des formules proposées est de l'ordre de 0,2%.

## EINE KORRELATION VAN "RECOVERY"-TEMPERATUR-DATEN FÜR ZYLINDER IN EINER KOMPRESSIBELEN STRÖMUNG BEI HOHEN REYNOLDS–ZAHLEN

**Zusammenfassung**—Eine Korrelation wird gemacht von "recovery" Faktor Daten vor kreisförmiger Zylinder von hoher thermischer Leitfähigkeit in einem Bereich der Stagnations-Reynolds–Zahl von $10^2 < Re < 10^4$ und Mach'sen Kenngrössen $0 < M < 5$. Der "recovery" Faktor wird abhängig gefunden von der Reynolds Zahl im subsonem Gebiet und die Daten zeigen Diskontinuitäten im transonem Bereich. Für den subsonen und den supersonen Bereich werden die zugehörigen Gleichungen angegeben. Ferner wird eine allgemeinere Formel für die beide Gebiete vorgestellt. Die Genauigkeit dieser Formeln ist zirka 0.2%.

## ОБОБЩЕНИЕ ДАННЫХ ПО ТЕМПЕРАТУРЕ ВОССТАНОВЛЕНИЯ ДЛЯ ЦИЛИНДРОВ В ПОТОКЕ СЖИМАЕМОЙ ЖИДКОСТИ ПРИ БОЛЬШИХ ЗНАЧЕНИЯХ ЧИСЛА РЕЙНОЛЬДСА

**Аннотация** — Проведено обобщение данных по коэффициенту восстановления для круглых цилиндров с высокой теплопроводностью в диапазонах значений числа Рейнольдса $10^2 < Re_0 < 10^4$ и значений числа Маха $0 < M < 5$. Найдено, что в дозвуковой области течения коэффициент восстановления зависит от числа Рейнольдса. В трансзвуковой области наблюдаются разрывы. Представлены эмпирические формулы отдельно для дозвуковой и сверхзвуковой областей и общая формула для диапазонов. Точность предложенных формул составляет 0,2%.